IX. *A Discourse concerning the Proportional Heat of the Sun in all Latitudes, with the Method of collecting the same, as it was read before the Royal Society in one of their late Meetings. By* E. Halley.

THere having lately arisen some Discourse about that part of the Heat of Weather, simply produced by the Action of the Sun; and I having affirmed, that if that were considered, as the only Cause of the Heat of the Weather, I saw no reason, but that under the Pole the solstitial Day ought to be as hot as it is under the Equinoctial, when the Sun comes vertical, or over the Zenith: for this reason, that for all the 24 Hours of that Day under the Pole, the Sun's Beams are inclined to the Horizon, with an Angle of 23 ½ degrees; and under the Equinoctial, though he come vertical, yet he shines no more then 12 Hours, and is again 12 Hours absent, and that for 3 Hours 8 Min. of that 12 Hours he is not so much elevated as under the Pole; so that he is not 9 of the whole 24 higher than 'tis there, and is 15 Hours lower. Now the simple Action of the Sun is, as all other Impulses or Stroaks, more or less forceable, according to the *Sinus* of the Angle of Incidence, or to the Perpendicular let fall on the Plain, whence the Vertical Ray (being that of the greatest Heat) being put *Radius*, the force of the Sun on the Horizontal Surface of the Earth will be to that, as the *Sinus* of the Sun's Altitude at any other time. This being allowed for true, it will then follow, that the time of the continuance of the Sun's shining being taken for a *Basis*, and the *Sines* of the Sun's Altitudes erected thereon as Perpen-

Perpendiculars, and a Curve drawn through the Extremities of thoſe Perpendiculars, the *Area* comprehended ſhall be proportionate to the Collection of the Heat of all the Beams of the Sun in that ſpace of time. Hence it will follow, that under the Pole the Collection of all the Heat of a tropical Day, is proportionate to a Rectangle of the *Sine* of 23½ *gr.* into 24 Hours or the Circumference of a Circle; that is; the *Sine* of 23½ *gr.* being nearly 4 tenths of Radius; as $\frac{8}{10}$ into 12 Hours. Or the Polar Heat is equal to that of the Sun continuing 12 Hours above the Horizon, at 53 *gr.* height, than which the Sun is not 5 Hours more elevated under the Equinoctial.

But that this matter may be the better underſtood, I have exemplied it by a Scheme (*Fig.* 8.) wherein the *Area ZG HH,* is equal to the *Area* of all the *Sines* of the Sun's Altitide under the Equinoctial, erected on the reſpective Hours from Sun-riſe to the Zenith; and the *Area* ♋ *H H* ♋ is in the ſame proportion to the Heat for the ſame 6 Hours under the Pole on the Tropical Day; and ⊙ *H HQ,* is proportional to the collected Heat of 12 Hours, or half a Day under the Pole, which ſpace ⊙ *H HQ* is viſibly greater than the other *Area HZGH,* by as much as the *Area HGQ* is greater than the *Area Z G* ⊙; which, that it it ſo, is viſible to ſight, by a great exceſs; and ſo much in proportion does the Heat of the 24 Hours Sun ſhine under the Pole, exceed that of the twelve Hours under the Equinoctial: whence *Cæteris paribus*, it is reaſonable to conclude, that were the Sun perpetually under the Tropick, the Pole would be at leaſt as warm, as it is now under the Line it ſelf.

But whereas the Nature of Heat is to remain in the Subject, after the Cauſe that heated is removed, and particularly in the Air; under the Equinoctial the twelve Hours abſence of the Sun does very little ſtill the Motion impreſs'd by the paſt Action of his Rays wherein Heat conſiſts, before he ariſe again: But under the Pole the long abſence of the ☉ for 6 Months, wherein the extremity of Cold does obtain, has ſo chill'd the Air, that it is as it were froſen, and cannot, before the Sun has got far towards it, be any way ſenſible of his preſence, his Beams being obſtructed by thick Clouds, and perpetual Foggs and Miſts, and by that *Atmoſphere* of *Cold*, as the late Honourable Mr. *Boyle* was pleaſed to term it, proceeding from the everlaſting Ice, which in immenſe Quantities does chill the neighbouring Air, and which the too ſoon retreat of the Sun leaves unthawed, to encreaſe again, during the long Winter that follows this ſhort Interval of Summer. But the differing Degrees of Heat and Cold in differing Places, depend in a great meaſure upon the Accidents of the Neighbourhood of high Mountains, whoſe height exceedingly chills the Air brought by the Winds over them; and of the nature of the Soyle, which variouſly retains the Heat, particularly the Sandy, which in *Africa*, *Arabia*, and generally where ſuch Sandy Deſarts are found, do make the Heat of the Summer incredible to thoſe that have not felt it.

In proſecution of this firſt Thought, I have ſolved the Problem generally, *viz.* to give the proportional degree of Heat or the ſum of all the *Sines* of the Sun's Altitude, while he is above the Horizon in any oblique Sphere, by reducing it to the finding of the Curve Surface of a Cylindrick Hoof, or of a given part thereof.

Now this Problem is not of that difficulty as appears at firſt ſight, for in *Fig.* 9. let the Cylinder ABCD be cut obliquely with the Ellipſe BKDI, and by the centre thereof H, deſcribe the Circle IKLM; I ſay, the Curve Surface IKLB is equal to the Rectangle of IK and BL, or of HK and 2 BL or BC: And if there be ſuppoſed another Circle, as NQPO, cutting the ſaid Ellipſe in the

points

points P, Q; draw PS, QR, parallel to the Cylinders Axe, till they meet with the aforesaid Circle IKLM in the points R, S, and draw the Lines RTS, QVP bisected in T and V. I say again, that the Curve Surface RMSQDP is equal to the Rectangle of BL or MD and RS, or of 2 BL or AD and ST or VP; and the Curve Surface QNPD is equal to RS $\times$ MD — the Arch RMS $\times$ SP, or the Arch MS $\times$ 2 SP: or it is equal to the Surface RMSQDP, subtracting the Surface RMSQNP. So likewise the Curve Surface QBPO is equal to the sum of the Surface RMSQDP or RS $\times$ MD, and of the Surface RLSQOP or the Arch LS $\times$ 2 SP.

This is most easily demonstrated from the consideration, That the Cylindrick Surface IKLB is to the inscribed Spherical Surface IKLE, either in the whole or in its Analogous Parts, as the tangent BL is to the Arch EL, and from the Demonstrations of *Archimedes de Sphæra & Cylindro, Lib.* I. *prop.* XXX. and XXXVII. XXXIIX. which I shall not repeat here, but leave the Reader the pleasure of examining it himself; nor will it be amiss to consult Dr. *Barrows*'s Learned Lectures on that Book, Published at *London*, *An.* 1684, *viz. Probl.* IX. and the Corollaries thereof.

Now to reduce our Case of the Sum of all the *Sines* of the Suns Altitude in a given Declination and Latitude to the aforesaid Problem, let us consider *Fig.* 10. which is the *Analemma* projected on the Plain of the *Meridian*, Z the Zenith, P the Pole, HH the Horizon, æ æ the Equinoctial, ♋ ♋, ♑ ♑ the two Tropicks, ♋I the *Sine* of the Meridian Altitude in ♋; and equal thereto, but perpendicular to the Tropick, erect ♋1, and draw the Line T 1 intersecting the Horizon in T, and the hour Circle of 6, in the Point 4, and 6 4 shall be equal to 6R, or to the *Sine* of the Altitude at 6: and the like for any other Point in the Tropick, erecting a Perpendicular thereat, terminated by the Line T 1: Through the Point 4 draw

draw the Line 4 5 7 parallel to the Tropick, and repreſenting a Circle equal thereto; then ſhall the Tropick ♋ ♋ in *Fig.* 10. anſwer to the Circle NOPQ, in *Fig.* 9. the Circle 4 5 7 ſhall anſwer the Circle IKLM, T 4 1 ſhall anſwer to the Elliptick Segment QIBKP, 6 R or 6 4 ſhall anſwer to SP, and 5 1 to BL, and the Arch ♋ T, to the Arch LS, being the ſemidiurnal Arch in that Latitude and Declination; the *Sine* whereof, though not expreſſible in *Fig.* 10. muſt be conceived as Analogous to the Line TS or UP in *Fig.* 9.

The Relation between theſe two Figures being well underſtood, it will follow from what precedes, That, *the Sum of the* Sines *of the Meridian Altitudes of the Sun in the two Tropicks, (and the like for any two oppoſite Parallels) being multiplied by the* Sine *of the ſemidiurnal Arch, will give an* Area *analogous to the Curve Surface R M S Q D P; and thereto adding in Summer, or ſubſtracting in Winter, the product of the length of the ſemidiurnal Arch, (taken according to* Van Ceulen's *Numbers) into the difference of the above-ſaid* Sines *of Meridian Altitude: the Sum in one caſe, and difference in the other ſhall be as the Aggregate of all the* Sines *of the Sun's Altitude, during his appearance above the Horizon; and conſequently of all his Heat or Action on the Plain of the Horizon in the propoſed Day.* And this may alſo be extended to the parts of the ſame Day; for if the aforeſaid Sum of the *Sines* of the Meridian Altitudes, be multiplied by half the Sum of the *Sines* of the Sun's horary diſtance from Noon, when the Times are before and after Noon; or by half their difference, when both are on the ſame ſide of the Meridian; and thereto in Summer, or therefrom in Winter, be added or ſubſtracted the product of half the Arch anſwerable to the propoſed Interval of Time, into the difference of the *Sines* of Meridian Altitudes, the ſum in one caſe, and diffe-

rence

rence in the other, ſhall be proportional to all the Action of the Sun during that ſpace of time.

I foreſee it will be objected, that I take the *Radius* of my Circle on which I erect my Perpendiculars always the ſame, whereas the Parallels of Declination are unequal; but to this I anſwer, that our ſaid circular Baſes ought not to be Analogous to the Parallels, but to the Times of Revolution, which are equal in all of them.

It may perhaps be uſeful to give an Example of the Computation of this Rule, which may ſeem difficult to ſome. Let the Solſtitial Heat, in ♋ and ♑ be required at *London*, *Lat.* 51°. 3′2.

38° - 2′8	*Co - Lat.*		*Diff. Aſcen.* 33° - 1′1.
23 - 30	*Decl.* ☉		*Arc.Semidi.æſtiv.* 123 - 11.
61 - 58	*Sinus* =	,882674	*Arc.Semidiur. hyb.* 56 - 49. *Sin.*,638923
14 - 58	*Sinus* =	,258257	*Arc.æſtiv.menſura* 2,149955.
	Summa	1,140931	*Arc. hyber menſura* 991683.
	Diff.	,624417	

Then 1,140931 in ,836923 + ,624417 in 2,149955 = 2,29734
And 1,140931 in ,836929 — ,624417 in ,991638 = 33895

So that 2,29734 will be as the Tropical Summers days Heat, and 0,33895 as the Action of the Sun in the Day of the Winter Solſtice.

After

After this manner I computed the following Table for every tenth Degree of Latitude, to the Equinoctial and Tropical Sun, by which an Eſtimate may be made of the intermediate Degrees.

Lat.	Sun in ♈ ♎	Sun in ♋	Sun in ♑
0	20000	18341	18341
10	19696	20290	15834
20	18794	21737	13166
30	17321	22651	10124
40	15321	23048	6944
50	12855	22991	3798
60	10000	22773	1075
70	6840	23543	000
80	3473	24673	000
90	0000	25055	000

Thoſe that deſire more of the Nature of this Problem, as to the Geometry thereof, would do well to compare the XIII *Prop. Cap.* V. of the Learned Treatiſe, *De Calculo Centri Gravitatis*, by the Reverend Dr. *Wallis*, Publiſhed *Anno.* 1670.

From this Rule there follow ſeveral Corollaries worth Note: As I. that the Equinoctial Heat when the Sun comes Vertical, is as twice the Square of *Radius*, which may be propoſed as a Standard to compare with in all other Caſes. II. That under the Equinoctial, the Heat is as the *Sine* of the Sun's Declination. III. That in the Frigid Zones when the Sun ſets not, the Heat is as the Circumference of a Circle into the *Sine* of the Altitude at 6. And conſequently that in the ſame Latitude theſe Aggregates of Warmth, are as the *Sines* of the Sun's Declinations; and in the ſame Declination of *Sol*, they are as the *Sines* of the Latitudes, and generally they are as the *Sines* of the Latitudes into the *Sines* of Declination.

IV. That

IV. That the Equinoctial Days Heat is every where as the Co-ſine of the Latitude. V. In all places where the Sun ſets, the difference between the Summer and Winter Heats, when the Declinations are contrary, is equal to a Circle into the *Sine* of the Altitude at 6 in the Summer Parallel, and conſequently thoſe differences are as the *Sines* of Latitude into or multiplied by the *Sines* of Declination. VI. From the Table I have added, it appears that the Tropical Sun under the Equinoctial has of all others the leaſt Force. Under the Pole it is greater than any other days Heat whatſoever, being to that of the Equinoctial as 5 to 4.

From the Table and theſe Coralleries may a general *Idea* be conceived of the Sum of all the Actions of the Sun in the whole Year, and that part of Heat that ariſes ſimply from the Preſence of the Sun be brought to a Geometrical Certainty: And if the like could be perform'd for Cold; which is ſomething elſe than the bare Abſence of the Sun, as appears by many Inſtances, we might hope to bring what relates to this part of *Meteorology* to a perfect Theory.

Fig. 2.

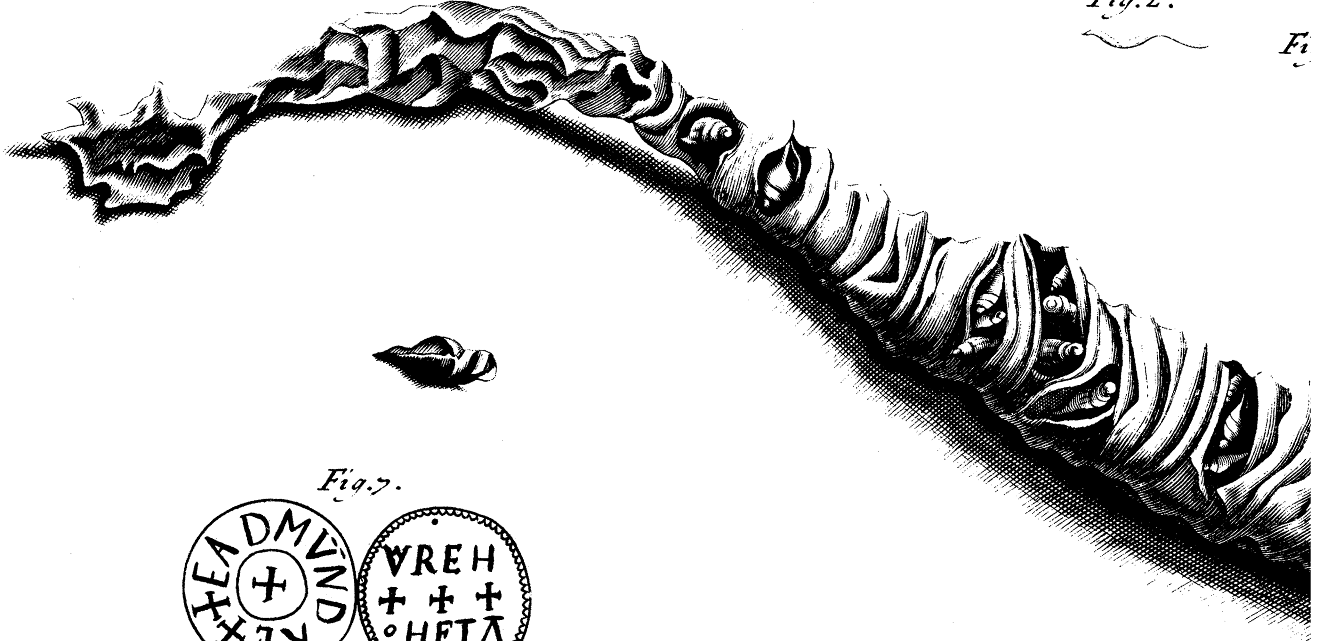


Fig. 7.

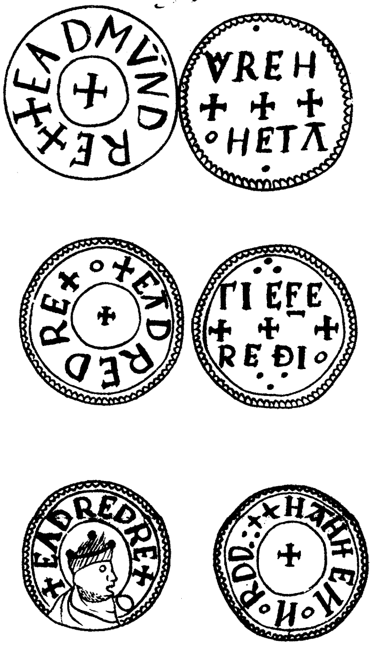
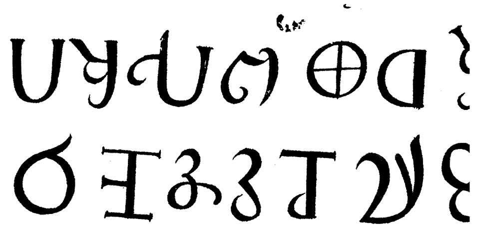


Fig. 6.



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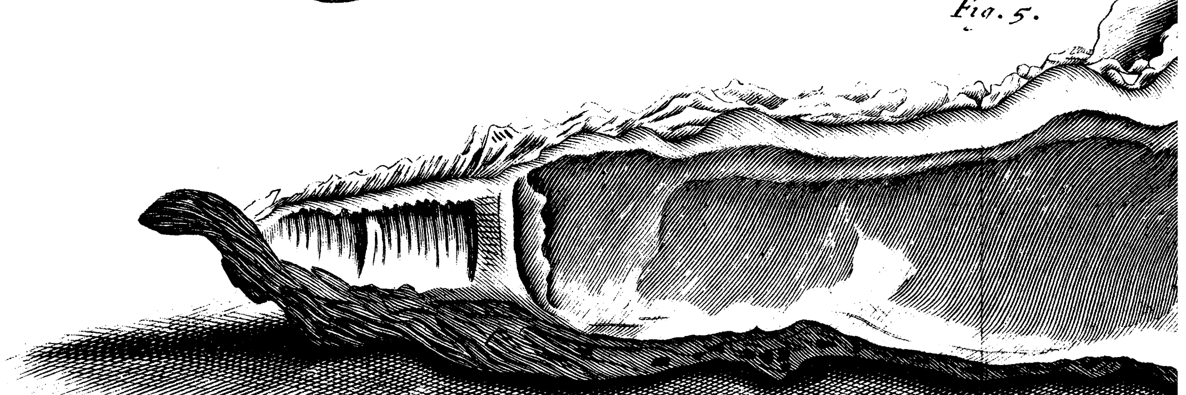

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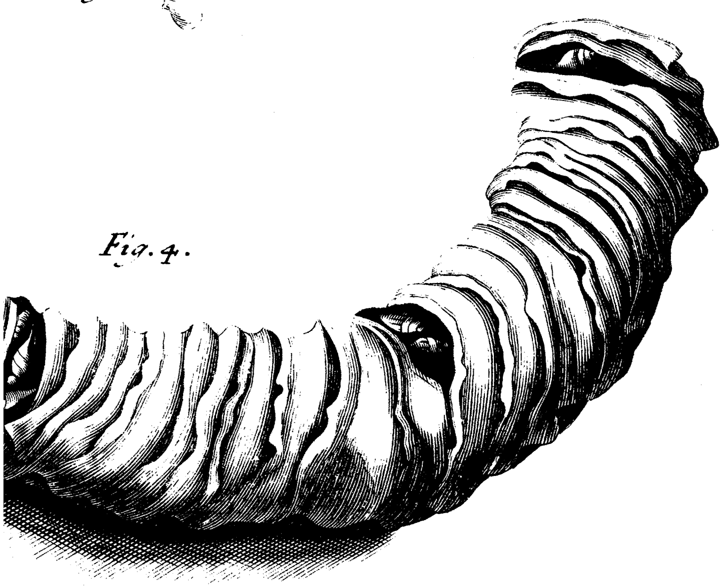
Fig.1.

Fig.3.

Fig.4.



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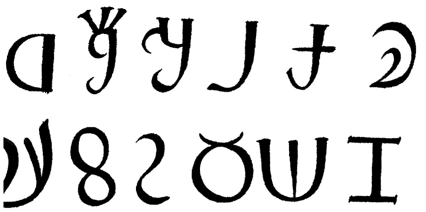


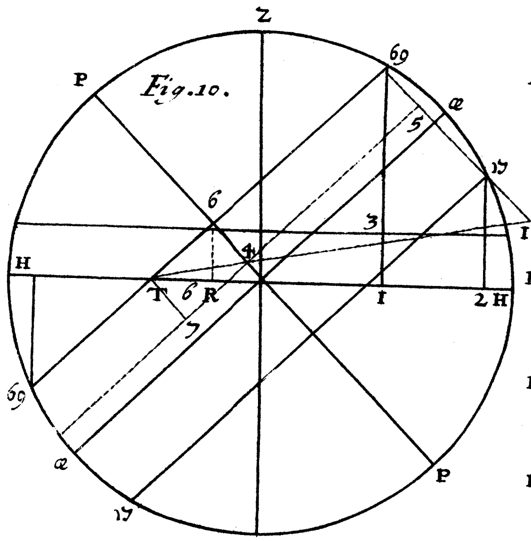


Fig. 10.

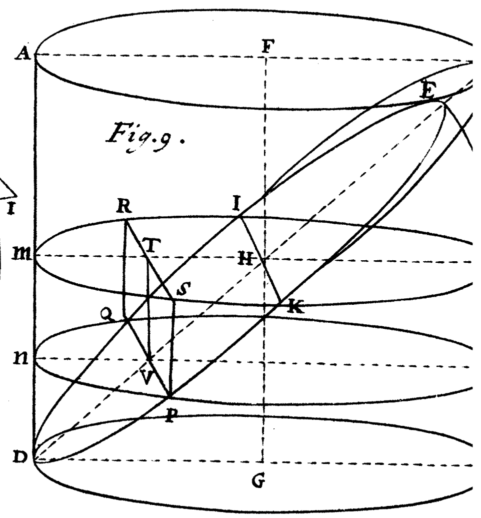
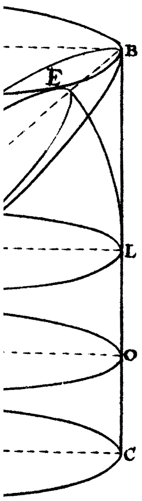


Fig. 9.



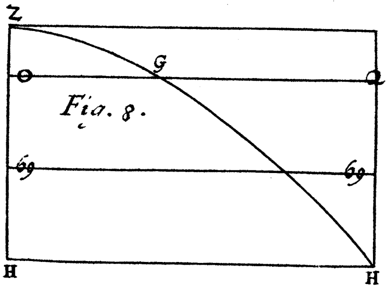


Fig. 8.

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 7.

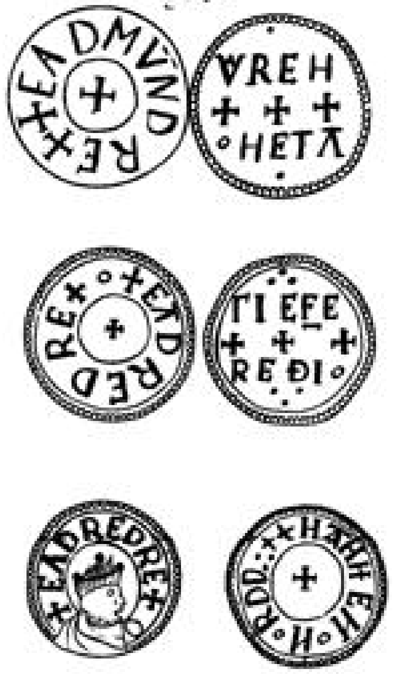


Fig. 6.

Fig. 5.

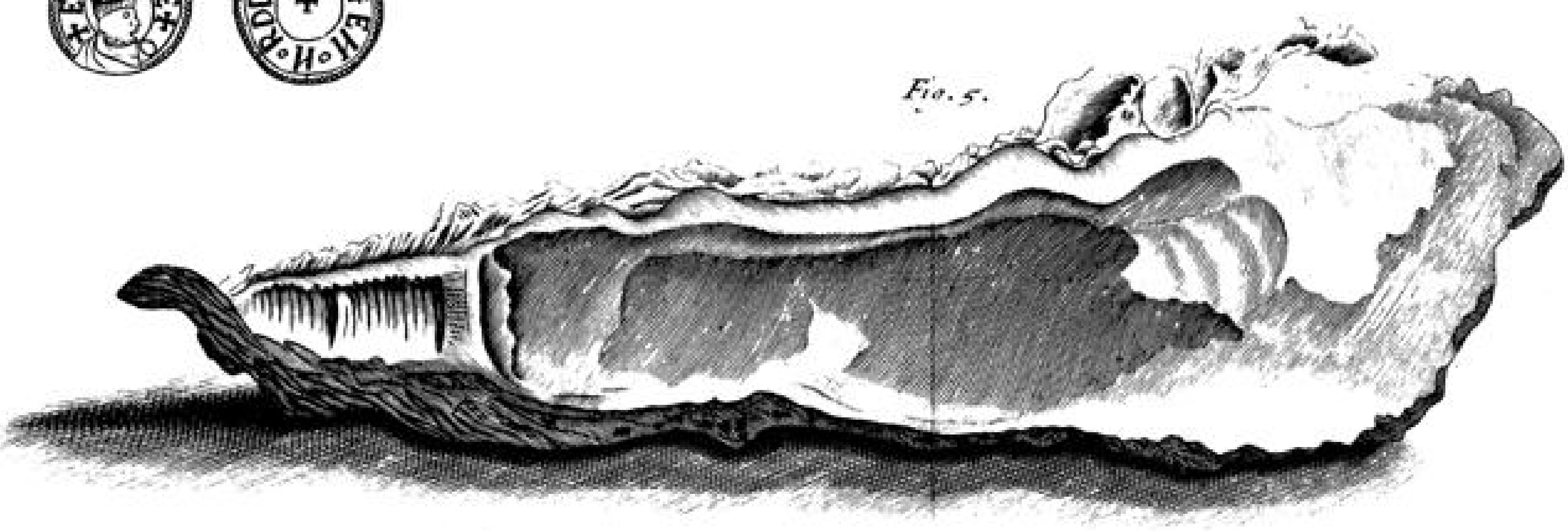


Fig. 10.

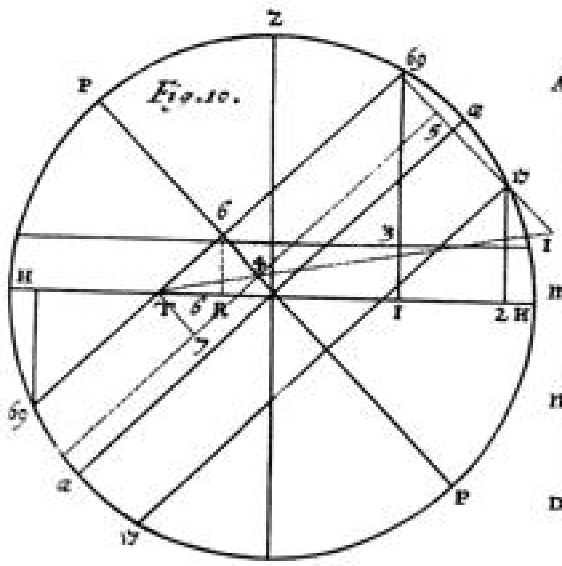


Fig. 9.

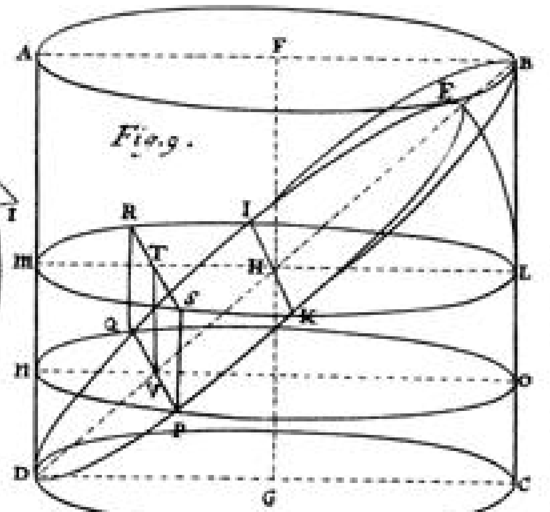


Fig. 8.